Instructions: Choose EXACTLY 10 points to be graded. I will evaluate anything you turn in but you MUST indicate which 10 points you want to count towards your grade. You will not earn full credit if your choice is not clear even if all of your solutions are correct. You are free to ask me any questions and work with whomever you like. If you work with any one else or use any resources beyond the textbook and class notes, please let me know. *Quiz 9 in D2L.*

For computations: you do not need to obtain a single value as your answer—it is acceptable to submit an expression that could be plugged into a scientific calculator and give a value. That is, all integrals must be evaluated but not necessarily simplified beyond that.

1. (4 points) Let f be a function with Taylor series given by

$$\sum_{n=0}^{\infty} c_n(x-2)^n$$

   and suppose that the radius of convergence is $R = 5$.

   (a) Where is the Taylor series centered and what can you say about the interval of convergence?

   (b) Recall that the coefficients of a Taylor series are given by the formula:

   $$c_n = \frac{f^{(n)}(a)}{n!}$$

   where $f^{(n)}(a)$ is the nth derivative of f evaluated at the center of the Taylor series, a. If $c_{13} = 104$, what is $f^{(13)}(2)$?

   (c) If possible, write a series for $f(0)$. If it is not possible, state why not.

   (d) If possible, write a series for $f(8)$. If it is not possible, state why not.

2. (2 points) Write a Maclaurin series for $f(x) = x\sin(2x)$. What is the interval of convergence?

3. (2 points) Write a Maclaurin series for $f(x) = \dfrac{x^4}{1+x}$.

4. (2 points) Express $\displaystyle\int \frac{e^x}{x}\,dx$ as an infinite series.

5. (2 points) Use series to approximate $\displaystyle\int_0^1 \sqrt{1+x^4}\,dx$ correct to two decimal places.

6. (2 points) Use series to evaluate the following limit:

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}$$

7. (1 point) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

(a) 1

(b) $\dfrac{\sqrt{3}}{2}$

(c) $\dfrac{1}{2}$

(d) Does not exist

8. (2 points) Give the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{5^n}{2n}(2x-1)^{n+1}$

9. (1 point) Which function is a solution to the differential equation $y'' + y = 2\cos(x)$?

   (a) $y = \cos(2x)$

   (b) $y = \sin(x^2)$

   (c) $y = x\sin(x)$

   (d) $y = \cos(x) - \sin(x)$

10. (1 point) Which function is a solution to the differential equation $y = \left(\dfrac{y''}{6}\right)^3$?

   (a) $y = (x-2)^3$

   (b) $y = 6e^{3x}$

   (c) $y = x^3 + x$

   (d) $y = 6x - 2$

11. (2 points) Solve the differential equation $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ given the initial condition $y(0) = -1$.

12. (2 points) A tank contanis 1000L of pure water. Brine (salt-water mixture) that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min/ The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank after t minutes? How much salt is in the tank after 1 hour?